**Structures and Interpretation of Computer Program**

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**Exercise 1.2.1**

**1.9. Evolution of process**

First procedure:

(+ 4 5 )  
(inc ( + ( dec 4 ) 5 ) )  
(inc ( inc ( + ( dec 3 ) 5 ) ) )  
(inc ( inc ( inc ( + ( dec 2 ) 5 ) ) ) )  
(inc ( inc ( inc ( inc ( + ( dec 1 ) 5 ) ) ) ) )  
(inc ( inc ( inc ( inc ( + 0 5 ) ) ) ) ) )  
(inc ( inc ( inc ( inc ( 5 ) ) ) ) )  
(inc ( inc ( inc ( 6 ) ) ) ) )  
(inc ( inc ( 7 ) ) ) )   
(inc ( 8 ) )   
(9)  
9

This is a recursive process since its expand first and then evaluate its deferred operation. The computer keep track of status of the operation before evaluating.

Second procedure:

(+ 4 5)  
(+ (dec 4 ) ( inc 5 ) )  
(+ ( dec 3 ) ( inc 6 ) )   
(+ ( dec 2 ) ( inc 7 ) )  
(+ ( dec 1 ) ( inc 8 ) )  
(+ ( 0 ) ( 9 ) )  
9

This is an iterative process because each step contains fixed state variable.

**1.10. Ackermann’s Function**

a. ( A 1 10)  
( A (- x 1) (A x (- y 1 ))))  
( A (- 1 1) (A 1 (- 10 1 ))))  
( A 0 (A 1 9)))  
( A 0 (( A (- 1 1) (A 1 (- 9 1 )))))))  
( A 0 (( A 0 (A 1 8))))))  
( A 0 (( A 0 (( A (- 1 1) (A x (- 8 1 ))))))))))  
( A 0 (( A 0 ( A 0 (A 1 7)))))))  
( A 0 (( A 0 ( A 0 ( A 0 (A 1 6))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 1 5)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0 A 1 4)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 1 3)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 0( A 0( A 1 2)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 0( A 0( A 0( A 1 1)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 0( A 0( A 0 2)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 0( A 0 4)))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0( A 0 8))))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0( A 0 16)))))))  
( A 0 (( A 0 ( A 0 ( A 0 ( A 0 32))))))  
( A 0 (( A 0 ( A 0 ( A 0 64)))))  
( A 0 (( A 0 ( A 0 128))))  
( A 0 (( A 0 256)))  
( A 0 512)  
1024

b. (A 2 4)  
(A 2 4)  
( A ( - 2 1 ) ( A ( 2 (- 4 1 ) ) ) )  
( A ( 1 ) ( ( A ( - 2 1 ) ( A ( 2 (- 3 1 ) ) ) )) )  
( A ( 1 ) ( ( A 1 ( A ( 2 2 ) ) )) )  
( A ( 1 ) ( ( A 1 ( ( A ( - 2 1 ) ( A ( 2 (- 2 1 ) ) ) )) )) )  
( A ( 1 ) ( ( A 1 ( ( A 1 ( A 2 1 ) )) )) )  
( A ( 1 ) ( ( A 1 ( ( A 1 2 ) )) )) )  
( A ( 1 ) ( ( A 1 ( ( A 0 ( A ( 1 1 ) ) ))) )) )  
( A ( 1 ) ( ( A 1 ( ( A 0 2 ) ))) )) )  
( A ( 1 ) ( ( A 1 ((\* 2 2 ))) )) )  
( A ( 1 ) ( ( A 1 4 ) ) )   
( A ( 1 ) ( ( ( A ( - 1 1 ) ( A ( 1 (- 4 1 ) ) ) )) ) )  
( A ( 1 ) ( ( ( A 0 ( A ( 1 3 ) ) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 3 1 ) ) ) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 ( A ( 1 2 ) ) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 2 1 ) ) ) )) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 ( ( A 0 ( A 1 1 ) )) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 ( ( A 0 2 )) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 ( \* 2 2) )) )) ) )  
( A ( 1 ) ( ( ( A 0 ( ( A 0 4 )) )) ) )  
( A ( 1 ) ( ( ( A 0 8) )) ) )  
( A 1 16 )   
( A ( - 1 1 ) ( A ( 1 (- 16 1 ) ) ) )  
( A 0 ( A ( 1 15 ) ) )  
( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 15 1 ) ) ) ) ) )  
( A 0 ( ( A 0 ( A ( 1 14 ) ) ) ) )  
( A 0 ( ( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 14 1 ) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( A ( 1 13 ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 13 1 ) ) ) )) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A ( - 1 1 ) ( A ( 1 (- 12 1 ) ) ) )) )) ) ) ) ) )

( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A ( 1 11 ) ) )) )) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A ( 1 10 ) ) ) ) )) )) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A ( 1 9 ) ) )) ) ) )) )) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A ( 1 8 ) ) )) )) ) ) )) )) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A ( 1 7 ) ) )) )) ) ) )) )) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A ( 1 6 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A ( 1 5 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A ( 1 4 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A ( 1 3 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A ( 1 2 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A ( 1 1 ) ) )) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( 2)) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 4) )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 8 )) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 ( A 0 16) ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 ( A 0 32 ) ) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 ( A 0 64) )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( A 0 128 )) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 256) )) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 512 ) ) ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 1024 ) ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 ( ( A 0 2048 ) ) ) ) )  
( A 0 ( ( A 0 ( ( A 0 ( ( A 0 4096 ) ) ) )  
( A 0 ( ( A 0 16384 ) )  
( A 0 32768)  
65536

Original Answer: 32768, Revised answers: 65536

c. A ( 3 3 )  
( A ( - 3 1 ) ( A 3 ( - 3 1 ) ) )  
( A ( 2 ) ( A 3 2 ) )  
( A ( 2 ) ( ( A ( - 3 1 ) ( A 3 ( - 2 1 ) ) ) )  
( A ( 2 ) ( ( A ( 2 ) ( A 3 1 ) ) ) )  
( A ( 2 ) ( ( A ( 2 ) ( 2 ) ) ) )  
( A ( 2 ) ( ( A ( - 2 1 ) ( A 2 ( - 2 1 ) ) ) ) )  
( A ( 2 ) ( ( A ( 1 ) ( A 2 1 ) ) ) ) )  
( A ( 2 ) ( ( A 1 2 ) ) ) )  
( A ( 2 ) ( ( A ( - 1 1 ) ( A 1 (- 2 1 ) ) ) ) ) )  
( A ( 2 ) ( ( A ( 0 ) ( A 1 ( 1 ) ) ) ) ) )  
( A ( 2 ) ( ( A ( 0 ) ( A 1 1 ) ) ) ) )  
( A ( 2 ) ( ( A ( 0 ) 2 ) ) ) )  
( A ( 2 ) ( ( A 0 2 ) ) ) )  
( A ( 2 ) ( ( \* 2 2 ) ) ) )  
( A 2 4 )  
…  
65536 from b.

d. Mathematical notation

d1. f (n) = 2n since (A 0 n) = ( \* 2 n )

d2.

Original Answer: X

g (n) = 0, if n=0  
 2, if n=1  
 2 (g (n-1) ) otherwise

Revised Answer:

if n = 0

(A 1 0)  
0

if n = 1

( A ( - 1 1 ) ( A ( 1 (- 1 1 ) ) ) )  
( A 0 ( A ( 1 0 ) ) )  
( A 0 0 )  
0

if n = 2  
( A ( - 1 1 ) ( A ( 1 (- 2 1 ) ) ) )  
( A 0 ( A ( 1 1 ) ) )  
( A 0 2 )  
4

if n = 3  
( A ( - 1 1 ) ( A ( 1 (- 3 1 ) ) ) )  
( A 0 ( A ( 1 2 ) ) )  
( A 0 ( A ( - 1 1 ) ( A ( 1 (- 2 1 ) ) ) ) )  
( A 0 ( A 0 ( A ( 1 1 ) ) ) )  
( A 0 ( A 0 2 ) )  
8

Thus, by mathematical induction, we can deduce:

g (n) = 0 if n = 0  
2^n if n > 0

d3.

h(n) = 0 if n=0  
2 if n=1  
2^(h(n-1)) if n >1

if

n = 1, 2^1 = 2

n = 2, 2^2 = 2^2 = 4

n = 3, 2^4 = 16

n = 4, 2^16 = 65536

All the answer get answer from previous term. Therefore 2^(h(n-1)).